Factoring Summary

Before factoring any polynomial, write the polynomial in descending order of one of the variables. Then note how many terms there are, and proceed by using one or more of the following techniques.

1. **ALWAYS** Factor out the Greatest Common Factor (GCF) first. Look for this in **<u>every</u>** problem. This includes factoring out the negative sign if it precedes the leading term.

   Example: $-x^2+6x-3 \ = \ -1(x^2-6x+3)$

   Example: $4x^3y^5 \ - \ 8x^2y^3 \ = \ 4x^2y^3(xy^2 \ - \ 2)$ where $4x^2y^3$ was the GCF.

2. If there are **<u>FOUR TERMS</u>**, try to factor by grouping (GR). Group two terms at a time, and factor out the greatest common factor from each group.

   Example:

   $x^3+6x^2-2x-12=$ *group the first two terms, then the last two terms*

   $x^2(x+6)-2(x+6)=$ *factor the* $(x+6)$ *out of both terms*

   $(x+6)(x^2-2)$ *this is the factored answer*

3. If there are **<u>TWO TERMS</u>**, look for one of these patterns:

   a. The difference of squares (DOS) factors into conjugate binomials (*conjugate means terms are separated by a plus sign in one binomial and a minus sign in the other binomial)*:

   $a^2-b^2=(a-b)(a+b)$

   Example: $9x^4-64y^2=(3x^2-8y)(3x^2+8y)$

   Note: a variable is a perfect square if the exponent is even

   b. The sum of squares does not factor: a^2+b^2 is prime (doesn't factor)

   Example: $9x^4 + 64y^2$ does not factor because it is the SUM of squares

   c. The sum of cubes (SOC) or difference of cubes (DOC) factors by these patterns: (each type contains a binomial times a trinomial)

   $a^3+b^3=(a+b)(a^2-ab+b^2)$

   Example: $(8x^3+27)=(2x+3)(4x^2-6x+9)$

   $a^3-b^3=(a-b)(a^2+ab+b^2)$

   Example: $(64x^6-125y^3)=(4x^2-5y)(16x^4+20x^2y+25y^2)$

   Note: a variable is a perfect cube if the exponent is a multiple of three

4. If there are **THREE TERMS**, look for these patterns:
    a. Quadratic trinomials of the form $ax^2 + bx + c \ \ where \ \ a = 1 \ (QT \ \ a = 1)$ factor into the product of two binomials (double bubble) where the factors of c must add to b.
        Example: $x^2 - 4x - 12 = (x - 6)(x + 2)$

    b. Quadratic trinomials of the form $ax^2 + bx + c \ \ where \ \ a \neq 1 \ (QT \ \ a \neq 1)$ eventually factor into the product of two binomials (double bubble), but you must first find the factors of *ac* that add to *b*, rewrite the original replacing *b* with these factors of *ac*, then factor by grouping to finally get to the double bubble.
        Example:
        $9x^2 + 15x + 4 \qquad ac = (9)(4) = 36$
        $factors \ \ of \ \ 36 \ \ that \ \ add \ \ to \ \ 15: \ \ 12 \ \ and \ \ 3$
        $\underline{9x^2 + 12x} \quad \underline{+3x + 4} = \ \ rewite \ 15x \ as \ 12x + 3x, \ then \ \ factor \ by \ grouping$
        $3x(x + 4) + 1(x + 4) =$
        $(x + 4)(3x + 1)$

    c. Quadratic square trinomials (*QST)* of the form $ax^2 + bx + c$ may factor into the square of a binomial. Look for the pattern where two of the terms are perfect squares, and the remaining term is twice the product of the square root of the squares:
        $a^2 \pm 2ab \pm b^2 = (a \pm b)^2$
        Example: $16x^2 - 40x + 25 = (4x - 5)^2$
        Note the pattern: Square root of $16x^2$ is $4x$. Square root of *25* is *5*.
        Twice the product of the square roots: *2(4x)(5)=40x*, which is the middle term

5. Factor all expressions completely. Sometimes, you will need to use two or three types of factoring in a single problem.
    Example: $-2x^4 + 32$

    $-2x^4 + 32 = \qquad factor \ \ out \ \ the \ \ GCF \ \ of \ \ -2$
    $-2(x^4 - 16) = \qquad factor \ \ the \ \ difference \ \ of \ \ squares$
    $-2(x^2 - 4)(x^2 + 4) = \qquad factor \ \ the \ \ remaining \ \ difference \ \ of \ \ squares$
    $-2(x - 2)(x + 2)(x^2 + 4) \qquad (remember \ \ that \ \ the \ \ sum \ \ of \ \ squares \ \ is \ \ prime)$