Solving Exponential and Logarithmic Equations

Exponential Equations (*variable in exponent position)*

1. Isolate the exponential portion ($base^{exponent}$): Move all non-exponential factors or terms to the other side of the equation.
2. Take *ln* or *log* of each side of the equation.
   - Make sure to use *ln* if the base is "e". Then remember that $\ln e = 1$.
   - Make sure to use *log* if the base is 10.
   - If the base is neither "e" nor "10", use either *ln* or *log,* your choice..
3. Bring the power (exponent) down into coefficient position.
4. Use various algebra techniques to solve for the variable.
5. Check your answer by evaluating the original equation with your calculator.

Example: $4e^{2x-3} = 40$ *Answer is on next page*

Logarithmic Equations

1. Move all log terms to one side of the equation, all non-log terms to the other side.
2. Combine log terms into a single log term using the laws of logarithms.
3. Write the log equation in its exponential form. (remember: $2^3 = 8 \leftrightarrow \log_2 8 = 3$)
4. Use various algebra techniques to solve for the variable.
5. Check your answer using your calculator. Remember that domain problems occur in log functions.
   - If the base of the log is "10" or "e", you can use the appropriate calculator keys.
   - If the log is not "10" or "e", you may need to use the change of base formula before using your calculator.

Example: $\log_5(x+1) - 2 = \log_5(x-1)$ *Answer is on next page*

Solving Exponential and Logarithmic Equations: Answers

Exponential Equation:

$4e^{2x-3} = 40$ Divide both sides by 4 to isolate the exponential portion

$e^{2x-3} = 10$ Take *ln* of both sides (use *ln* because the base is "*e*")

$\ln e^{2x-3} = \ln 10$ Use the law of logs that allows you to bring the power into coefficient position

$(2x-3)\ln e = \ln 10$ Recall that ln *e* is equivalent to 1, so the equation is actually $\rightarrow$

$2x - 3 = \ln 10$ Now just solve for x using algebra

$2x = 3 + \ln 10$ (add 3 to both sides, then divide both sides by 2)

$x = \dfrac{3 + \ln 10}{2} \approx 2.651$

Logarithmic equation

$\log_5(x+1) - 2 = \log_5(x-1)$ Move log terms to one side, non-log terms to other side

$\log_5(x+1) - \log_5(x-1) = 2$ Use laws of logs to write a single log expression

$\log_5\left(\dfrac{x+1}{x-1}\right) = 2$ Rewrite in exponential form

$5^2 = \dfrac{x+1}{x-1}$ Solve using algebra techniques

$25 = \dfrac{x+1}{x-1}$ Multiply both sides by $x-1$

$25(x-1) = x+1$ Use the distributive property to remove parenthesis

$25x - 25 = x + 1$ Subtract an *x* from both sides, add 25 to both sides

$24x = 26$ Divide by 24

$x = \dfrac{26}{24} = \dfrac{13}{12}$ Always reduce answers to lowest terms